c. The size of array is n = 2k

Base case: for k = 1, we have 2 numbers in the array. By line 1 - 4 in GATHER, we successfully set these two numbers. So, it is correct.

Inductive Hypothesis:

Assume: SLOWSORT and GATHER is correct for all k<= i size array. (i belongs to positive integer).

Prove: SLOWSORT is correct for array size: 2i+1, let’s say array A.

We equally divided A into two parts. After we perform SLOWSORT two times, we got two sorted arrays, whose size are both 2i. Let’s say this time A becomes A, C.

Then we equally divided these two parts into four parts, we called them A, B, C, D. And we have A<B, C<D, which means all the elements from A are smaller than all the element from B, similarly for C and D.

Now we need to prove GATHER[ABCD] can sort the array correctly.

1. Because length is greater than 2, then we perform swap first.

Step 6 – 7 swaps the middle of the array, we got ABCD -> ACBD.

1. Step 9 gathers the first two parts (GATHER[AC]). Then A and C becomes A’ and C’.

Because we assumed GATHER is correct for all k<=I size array, so now A’ and C’ are correctly sorted. Then A’ < C’, which means all the elements from A’ are smaller than all the elements from C’. Because A<B, C<D, and A’ currently contains half of the smallest elements of AC, and apparently not have elements which is bigger than A, then A’ < B, similarly A’ < D

1. Step 10 gathers the last two parts (GATHER[BD]). Then B and D becomes B’ and D’, similarly above, B’ and D’ are correctly sorted. Then B’ < D’. Because A’ < B, and A’ < D, so A’ < all the elements from BD then A’ < B’, A’ < D’. Because A < B then all the elements in A is smaller than B, C < D, then all the elements from C is smaller than D, then all the elements from A and C is smaller than D. And D’ is the biggest part of B and D, then C’ < D’.
2. Step 11 gathers the middle two parts (GATHER[C’B’]). Then C’ and B’ becomes C’’ and B’’. Then C’’ < D’’. And because A’ < C’, C’ < D’, then all the elements in A’ are smaller than C’ and D’, Then A’ < C’’. Because B’ < D’, C’ < D’, then all the elements in C’’ and B’’ are smaller than D’.

In conclusion, we proved that SLOWSORT AND GATHER holds for array size of 2i+1. Becuase A’ < C’’ < B’’ < D’ which means the algorithm correctly sorted the ABCD, which is size of 2i+1.